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A fundamental study of flow and heat transfer performances of downward water flow at low Reynolds numbers in a vertical heated straight tube

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Abstract—This paper is a report on fundamental experimental research to clarify performances of heat and fluid fields using water flow in a vertical heated straight circular tube in the Reynolds number region below 2000. Experiments are performed using a brass tube of 22 mm i.d. and 1500 mm in length which is electrically heated under constant heat flux conditions. Heat insulating layers are wrapped around the tube to prevent heat loss. It is made clear that under the smaller wall heat flux condition heat transfer deterioration is observed due to a decrease of the temperature gradient at the tube inner surface caused by the buoyancy force. It is also found that, even in the Reynolds number region of 500–800, in the case of large heat flux, heat transfer enhancement is observed due to the velocity fluctuation caused by flow instability due to the appearance of an inflection point in the velocity profile owing to the strong buoyancy force effect near the wall.

1. INTRODUCTION

THE FLOW in a straight circular tube, which is not heated, is a fundamental phenomenon in hydrodynamics and has been widely studied. On the other hand, when the tube is heated in the laminar flow region, owing to the difference between the directions of gravity and the tube axis, the flow is much different from the case of unheated tube flow, and a lot of research has been reported. Most reports on this research are concerned with heat transfer performances on horizontal and upward tube flows. In the former case, the secondary flow induced by buoyancy vertical to the tube axis enhances heat transfer performances [1]. Many discuss this flow [2] and recently numerical studies have been reported which consider the heating entrance region [3]. In the case of upward flows, the fluid near the heated inner tube surface is accelerated by buoyancy resulting in a swelling of the velocity profile near the wall from the Poiseuille one. This causes the enhanced heat transfer performance [4, 5]. In the case of downward flows, even though a few papers discuss cases of small heat flux at the tube surface, the case of large heat flux is numerically studied predicting heat transfer deterioration due to a decrease of the fluid temperature gradient at the surface [6, 7]. However, in this case, the fluid velocity gradient at the tube surface decreases due to buoyancy, which may introduce an inflection point in the velocity profile and then velocity fluctuation and cause enhancement of the wall heat flux due to fluctuating velocity. Few papers discuss cases of large heat flux in downward tube flows, particularly for clarifying phenomena related to the cases mentioned above.

The phenomena discussed in this paper are related essentially to hot-dry rock geothermal energy systems in which water is sent down in a vertical long circular well drilled in the underground rock and is heated by the hot wall of the well heated by magma existing deep in the ground. The hot-dry rock geothermal system is now under development not only in Japan but also in the U.S. and other countries. However, it has been reported that the water is not heated as much as expected though the reason for this is unclear.

In consideration of the situation, a fundamental study of heat transfer is made on downward flows in a vertical straight circular tube in the low Reynolds number region below 2000 and on the dependence of heat transfer performance on the heat flux along the tube wall. In order to make clear the reason of the heat transfer performance on heat flux, the velocity field is measured using an LDV device and discusssions are made on the relation of the velocity to heat transfer fields. A vertical straight circular tube of 22 mm i.d. and 1500 m in length is used in experiments. It is electrically heated from the outside and the temperatures at several wall points and that of liquid are measured by thermocouples. Based on results thus obtained heat transfer performances are widely discussed.

2. EXPERIMENTAL APPARATUS

The outside of a brass tube of 22 mm i.d. and 1.5 m in length is covered by thin insulating glass-wool

NOMENCLATURE

d	tube i.d., 22 mm	U	mean fluid velocity at r
Gr	local Grashof number, $g\beta(d/2)^3\Delta t/v^2$	U_m	mean velocity in a cross section
Nu	Nusselt number, $\alpha d/\lambda$	u	fluctuating axial velocity
Nu_0	Nusselt number without buoyancy effect	x	axial distance from tube inlet.
Pr	Prandtl number		
q	heat flux at tube wall		
r	radial direction coordinate		
R	tube radius, $d/2$	Greek symbols	
Re	Reynolds number, $U_m d/v$	α	heat transfer coefficient
T	temperature	β	volumetric expansion coefficient
Δt	temperature difference between tube wall and mixed mean fluid	λ	thermal conductivity
		v	thermal viscosity.

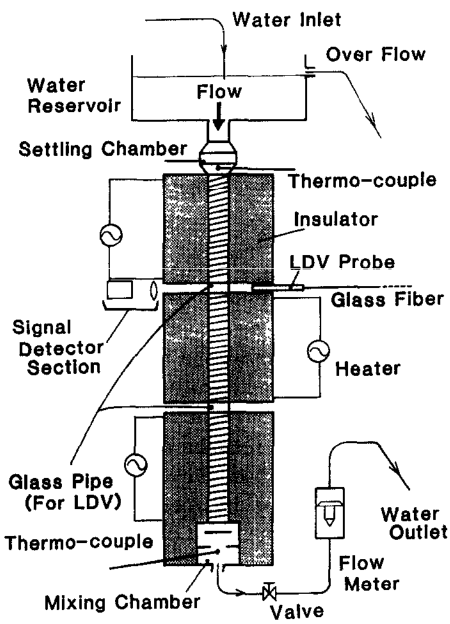

FIG. 1. System picture of the experimental apparatus.

tape and around it six nichrome wires are separately wound in series as to make the constant heat flux condition along the tube axis direction. Over the heating wire is laid a cover of glass-wool and rock-wool layers 65 mm thick to minimize the heat loss. Figure 1 shows the experimental apparatus which has a settling chamber at the top of the tube for decaying turbulence of inflowing water. The velocity is measured at two positions at 1/3 and 2/3 of the tube length from the top using an LDV device. A He–Ne laser is used and the LDV device is connected to the laser by an optical fiber. The flow field is considered to be axisymmetrical and the time-averaged and fluctuating velocities in the axis direction are measured.

The part for velocity measurement by the LDV device is made of glass tube of 24 mm i.d. and flanges. It is heated by electrical heaters to avoid heat loss. In this study, the velocity profile in the vicinity of the tube has to be accurately measured as the buoyancy effect is predominant in this area, so the laser light is modulated into 80.00 and 80.01 MHz using a Bragg cell. The dependence of the refractive index of water on temperature might cause changes of the angle and location of the beam intersection. However, in the experiments, the temperature field is considered to change in the radial direction and is not considered in the circumferential direction. The temperature distribution in the radial direction does not give effects on the beam intersection angle and location. Still less, the temperature changes in the radial direction are within 15°C which brings in an error of 10^{-3} only. Therefore, the dependence of the refractive index of water on the temperature in the experiments may be neglected. The LDV measuring device is mounted on a movable base sliding on a rigid stationary bed and is moved by a step-motor with a 0.010 mm movement for each step as shown in Fig. 2. Using this device, the LDV probe, leans and photo-multiplier are fixed on the base and moved in a body. Thus accurate measurements are performed.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

First, heat transfer experiments were made and then velocity measurements were performed using an LDV device. Original results from temperature measurements to calculate heat transfer performances are shown in Figs. 3–6. In these figures, the open circles indicate temperatures measured using the settling chamber at the inlet and the mixing chamber at the exit of the tube. On the other hand, the black circles show the wall temperatures measured by Cu–Co thermocouples welded on the brass tube.

3.1. *Experimental results of heat transfer performance*

Figure 3 shows an experimental result when the Reynolds number is low and heat flux is small, and a similar result is obtained also for $Re = 773$ and $q = 0.49\ \mathrm{kW\,m^{-2}}$. The broken line in Fig. 3 shows the

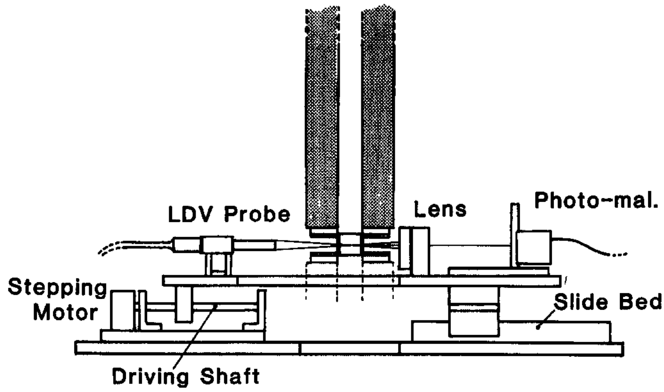

FIG. 2. Measurement part of the LDV device.

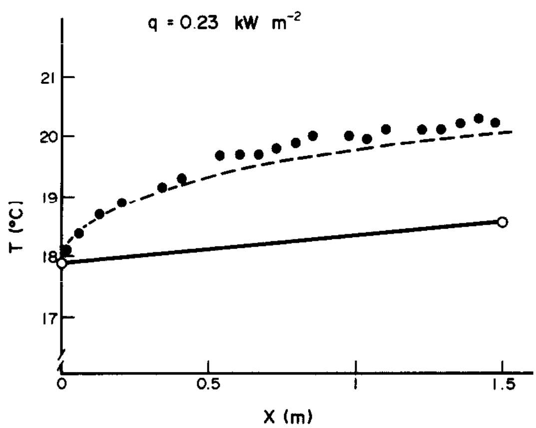

FIG. 3. Temperature distribution in the axial direction for a small Reynolds number and heat flux.

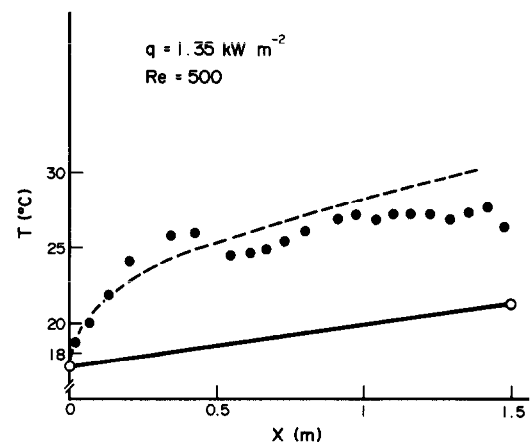

FIG. 5. Temperature distribution for a small Reynolds number and a moderately high heat flux.

value predicted by the Poiseuille flow profile. A result in the case of low Reynolds number and comparatively high heat flux is shown in Fig. 5 where downstream of the entrance region a deteriorated heat transfer performance is seen. Whereas, in Figs. 5 and 6, except for the entrance region, heat transfer performance is seen to be much enhanced with heat flux in comparison with the broken line showing no deterioration and enhancement. The local Nusselt number Nu is calculated from the solid line connecting

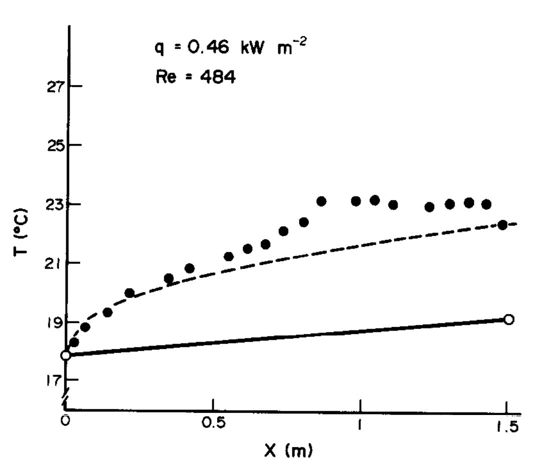

FIG. 4. Temperature distribution for a small Reynolds number and a slightly high heat flux.

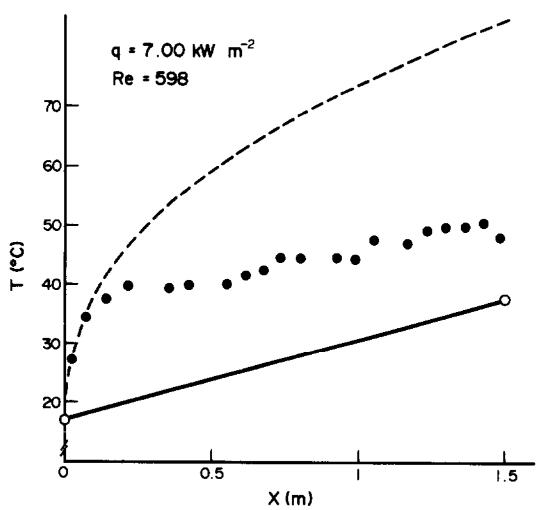

FIG. 6. Temperature distribution for a high heat flux.

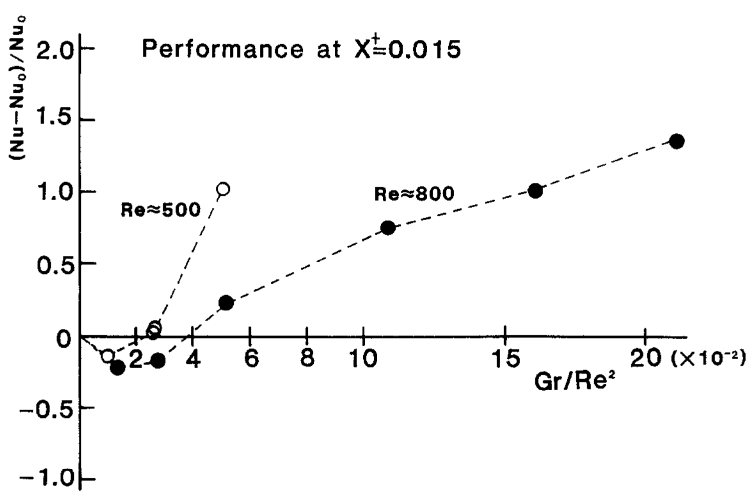

FIG. 7. Heat transfer performance change by Gr/Re^2.

the inlet and exit liquid temperatures and the local temperature, while the local Nusselt number for the Poiseuille flow is expressed by Nu_0. The enhancement factor is defined by the relation $(Nu-Nu_0)/Nu_0$. Based on many experimental results as shown in Figs. 3–6, the enhancement factors are plotted in Fig. 7 against Gr/Re^2 with Reynolds number as a parameter, where x^* is the non-dimensional axial distance defined by

$$x^* = (x/R)/(Pr\,Re) \tag{1}$$

where x is the distance from the tube entrance, that is the top of the tube. As seen from Fig. 7, for a given Reynolds number, with an increase in heat flux, that is an increase of Grashof number, heat transfer deterioration is seen first, and then heat transfer performance gives way to heat transfer enhancement. From this it would be understood that in the deterioration region the temperature gradient at the wall surface is reduced by the buoyancy force as the wall is at the highest temperature in a cross section. This is seen in the case when $Gr/Re^2 = 0.04$ for $Re = 800$. For $Gr/Re^2 > 0.04$, Nu becomes larger than Nu_0. In order to make the phenomenon clearer, the change of Nusselt number is shown in Fig. 8, the abscissa of which is Gr/Re^2 and Gr is the local Grashof number. The region below the abscissa is seen to deteriorate, while that above it indicates the enhanced heat transfer region. This figure shows that when $q < 1\ \mathrm{kW\,m^{-2}}$ in the low and medium Reynolds number regions the heat transfer performance deteriorates more in the downstream domain. On the other hand, when the heat flux is larger than $1\ \mathrm{kW\ m^{-2}}$ and the Reynolds number is above the medium value of 800, heat transfer performance is enhanced within a short distance and when the Reynolds number is as small as 400 it increases rather gradually. In the case of higher Reynolds number and larger heat flux, Gr/Re^2 is less dependent on x^* as the local temperature difference between the tube and the water is almost constant.

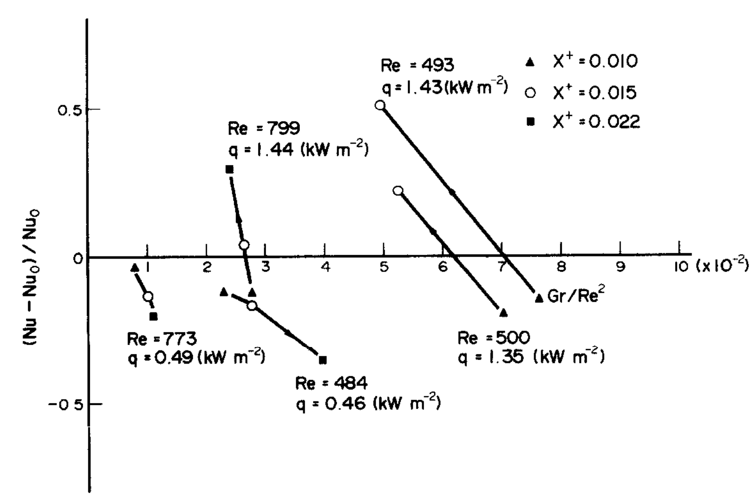

FIG. 8. Heat transfer performance change along the tube axis.

3.2. *Experimental results of velocity field and discussions on them*

In discussing the experimental results described above, it is considered that when heat transfer enhancement is observed the flow near the hot wall may not necessarily be laminar. Even though the critical Reynolds number of a flow is about 2000 in an unheated circular tube, when the tube is heated and buoyancy introduces a velocity opposing the original flow as discussed above, the following features of the velocity field may be seen.

(1) In the region near the hot tube surface, the influence of the buoyancy force is so strong as to introduce an inflection point in the velocity profile near the wall.

(2) In the region near the inflection point velocity instability is amplified even when the Reynolds number is much lower than 2000. The instability is more amplified in the downstream region.

In order to make sure of the prediction mentioned above, the LDV device was used to measure the mean and fluctuating velocities. For securing the accuracy of the LDV measuring method, the velocity profile in an unheated tube was measured to make sure that it coincides with the Poiseuille flow and the result thus obtained in Fig. 9 where the black circles indicate the mean values and the open circles the fluctuating velocity intensities. The figure proves a good accuracy of the LDV device for velocity measurement even in the region very close to the tube surface. It should be noted that even though the fluctuating velocity intensity is about 7%, it does give the least effect on the mean profile. Figure 10 shows the measured result at the time when the Reynolds number is near 800 and the heat flux is 0.49 kW m^{-2}. A slight dent is seen in the mean velocity profile near the region where $r/R = 0.8$ and the difference between the solid line expressing the experimental result of the mean velocity and the broken line expressing the Poiseuille profile is large at the center, while a small increase of the velocity fluctuation intensity $\sqrt{\langle u^2 \rangle}/U$ is seen near $r/R = 0.8$. As seen in Fig. 11, for a Reynolds number of about 800 and a heat flux of 1.44 kW m^{-2}, even though the mean velocity profile is the same as that of the Poiseuille flow in the upstream region, the mean velocity in the downstream region has a considerable dent near $r/R = 0.8$ and a large fluctuating velocity intensity exists particularly near the point of $r/R = 0.8$. The fluctuating intensity becomes larger further downstream. Velocity fluctuation is considered to be caused at the inlet point in the mean velocity profile and the fluctuation diffuses in the cross section. In other words, even when the local Reynolds number is below 2000, due to the appearance of the velocity fluctuating amplified in the downstream direction the mean heat transfer is enhanced by the convective heat transfer based on velocity fluctuation.

Summing up the discussions above, for a given Reynolds number, when the heat flux is smaller than $q = 0.46$ kW m^{-2} and no velocity fluctuation occurs,

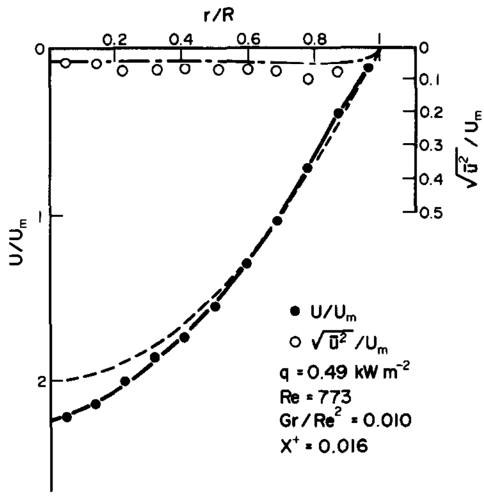

FIG. 10. Mean and fluctuating velocities in a moderately heated tube.

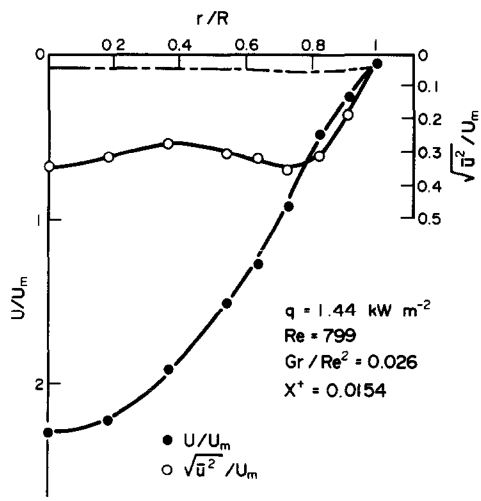

FIG. 11. Mean and fluctuating velocities in a highly heated tube.

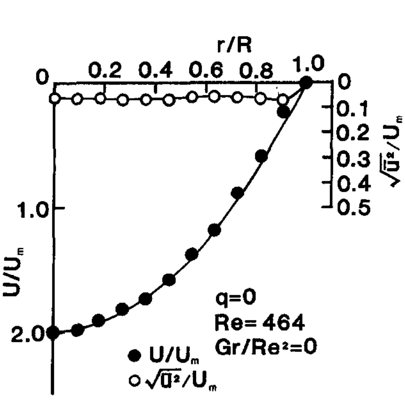

FIG. 9. Mean and fluctuating velocities without a heating tube.

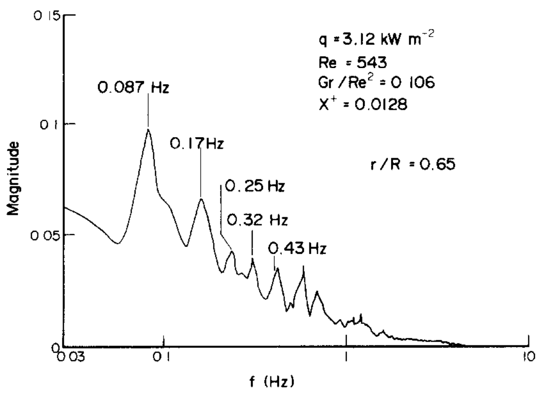

Fig. 12. Spectral of fluctuating velocity.

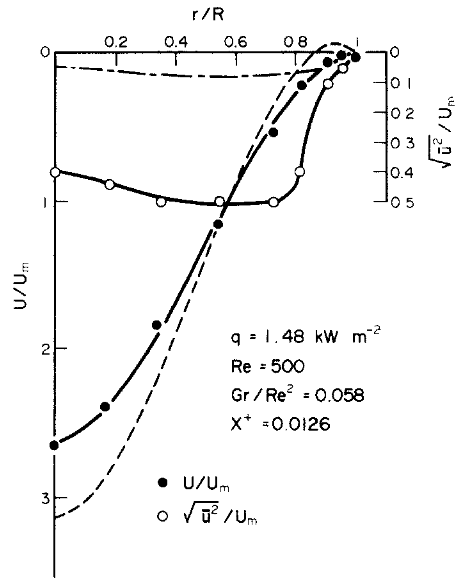

Fig. 13. Comparison between predicted and experimental values in the low Reynolds number and high heat flux case.

the temperature gradient at the wall decreases downstream of the tube. Thus Nusselt number decreases with x^* resulting in heat transfer deterioration as discussed in Fig. 8. On the other hand, when the heat flux is as large as 1.35 or 1.43 kW m^{-2}, heat transfer performance is enhanced even when it is deteriorated upstream.

In order to clarify the basic performance of enhanced heat transfer when the heat flux is large enough, the spectrum of a velocity fluctuation is taken and an example is shown in Fig. 12 where the scale of the ordinate is an arbitrary one. Figure 12 shows that when the Reynolds number is as low as 540, but the heat flux is high, the triple derivative of the velocity profile near its inflection point becomes very large and transition such as observed in natural free convection along a vertical plate may occur. The velocity fluctuation mentioned above is so amplified, mainly by the non-linear inertia term and others of the Navier–Stokes equation, as to excite higher harmonics of the transition wave. Figure 12 shows a spectrum at $r/R = 0.65$ where the velocity fluctuation intensity is highest. The spectrum has a pattern quite different from that of a turbulent flow, but has several peaks of higher harmonics excited by the fundamental transition wave. The wave and and its harmonics cause forced convection and those waves play a substantial role in heat transfer enhancement, but a theoretical analysis is needed to make the mechanism of heat transfer enhancement clearer.

As the Reynolds number treated in this experiment is below the critical Reynolds number of laminar flow, that is 2000, a comparison of our experimental results with future theoretical analyses is desirable. Hanratty *et al.* [6] reported a theoretical study of fully developed laminar downward flows in a vertical heated circular tube using the Boussinesq approximation for the buoyancy term. A comparison of our experiment and Hanratty *et al.*'s analysis is shown in Fig. 13 in the case of the same Reynolds number and heat flux. In this figure, the broken line shows the theoretically predicted value for the mean velocity profile and the solid line connecting black circles shows the experimental mean velocity profile obtained in this study. The lower solid line connecting the open circles is correlated so as to coincide with the experimental results of the velocity fluctuating intensity. The chain line indicates the intensity of the initial turbulence.

It is to be noted that even though the theoretical analysis predicts an adverse flow near the wall due to buoyancy force, experiments reveal that instead of occurrence of adverse flow the laminar flow condition is broken and the velocity fluctuation is brought about which results in the modification of the mean velocity profile. This comparison of the theoretical and experimental studies would definitely indicate an applicable limit of the theoretical study, and suggests the importance of the experimental study particularly on problems of a complicated phenomenon such as that taken up in this report.

The discussion made above leads to the following important future problems.

(1) The flow transition in a heated downward tube is different from that known for transition in an unheated flow, and the triple derivative of the mean velocity profile plays an important role in causing laminar transition at Reynolds numbers less than 2000. The transition wave rarely goes upward.

(2) As described in detail in this paper, the velocity fluctuation has a feature of having a spectrum consisting of transition waves, and transition to turbulent flow and occurrence of Reynolds stress should be studied in the future. In consideration of these facts, an expression of the velocity fluctuation is used instead of turbulence.

(3) It might be predicted that in the case of Reynolds numbers above 2000 heat deterioration is anticipated even in large heat flux cases.

4. CONCLUSION

Heat transfer performances of downward flows of Reynolds numbers below 2000 in a vertical straight circular heated tube were experimentally studied with flow field measurements using an LDV velocity meter and the following conclusions are obtained.

(1) In the case of a small heat flux, temperature and velocity gradients at the tube wall are decreased by the buoyancy force resulting in heat transfer deterioration.

(2) In the case of a large heat flux, the inflection point appears in the mean velocity profile near the wall caused by buoyancy and velocity fluctuation appears in the Reynolds number region above 500 resulting in heat transfer enhancement due to convection by velocity fluctuation.

(3) The velocity fluctuation effect caused by high heat flux rarely goes upstream and is a local feature.

(4) The spectrum of velocity fluctuation is different from those of turbulent flow but consists of fundamental and harmonic frequencies of the transition wave.

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ETUDE FONDAMENTALE DES PERFORMANCES D'ECOULEMENT ET DE TRANSFERT DE CHALEUR DE L'EAU DESCENDANT AUX FAIBLES NOMBRES DE REYNOLDS DANS UN TUBE VERTICAL, RECTILIGNE ET CHAUD

Résumé—On décrit une recherche expérimentale fondamentale pour clarifier les performances des champs de vitesse et de température de l'eau s'écoulant dans un tube circulaire droit et chauffé, pour des nombres de Reynolds inférieurs à 2000. Des expériences concernent un tube de laiton de 22 mm de diamètre intérieur et de 1500 mm de longueur, chauffé électriquement dans des conditions de flux constant. Des couches isolantes sont enroulées autour du tube pour éviter les pertes thermiques. Pour un flux thermique pariétal faible, on observe une détérioration du transfert de chaleur à cause de la décroissance du gradient de température sur la surface interne du tube du fait des forces de flottement. On trouve aussi que, même dans la région du nombre de Reynolds entre 500 et 800, dans le cas des grands flux thermiques, il y a un accroissement du transfert thermique à cause des instabilités dues à un point d'inflexion du profil de vitesse lié à une importante force de flottement près de la paroi.

GRUNDLEGENDE UNTERSUCHUNGEN VON STRÖMUNG UND WÄRMEÜBERGANG IN EINEM SENKRECHTEN BEHEIZTEN GERADEN ROHR BEI ABWÄRTSGERICHTETER WASSERSTRÖMUNG MIT KLEINER REYNOLDS-ZAHL

Zusammenfassung—In diesem Beitrag werden grundlegende experimentelle Untersuchungen des Temperatur- und Geschwindigkeitsfeldes in einem senkrechten beheizten geraden kreisrunden Rohr bei Reynolds-Zahlen unterhalb von 2000 beschrieben. Zur Durchführung der Versuche wurde ein 1500 mm langes Messingrohr mit einem Innendurchmesser von 22 mm benutzt, welchem durch eine elektrische Heitzung ein konstanter Wärmestrom aufgeprägt wurde. Zur Vermeidung von Wärmeverlusten wurde das Rohr isoliert. Es wird gezeigt, daß bei kleiner Wärmestromdichte an der Rohrwand eine Verschlechterung des Wärmeübergangs auftritt. Dies wird auf eine Abnahme des Temperaturgradienten an der inneren Rohroberfläche aufgrund des Auftriebs erklärt. Außerdem wird gezeigt, daß bei großer Wärmestromdichte im Bereich der Reynolds-Zahlen von 500 bis 800 eine Verbesserung des Wärmeübergangs auftritt. Aufgrund starker Auftriebskräfte nahe der Wandoberfläche kommt es zu einem Wendepunkt im Geschwindigkeitsprofil. Dies führt zu Strömungsinstabilitäten und in der Folge zu Fluktuationen der Strömungsgeschwindigkeit, wodurch der Wärmeübergang verbessert wird.

ФУНДАМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ГИДРОДИНАМИЧЕСКИХ И ТЕПЛООБМЕННЫХ ХАРАКТЕРИСТИК НИСХОДЯЩЕГО ПОТОКА ВОДЫ В ВЕРТИКАЛЬНОЙ НАГРЕВАЕМОЙ ПРЯМОЙ ТРУБЕ ПРИ МАЛЫХ ЧИСЛАХ РЕЙНОЛЬДСА

Аннотация—Представлены результаты фундаментального экспериментального исследования гидродинамических и тепловых полей при течении воды в вертикальной нагреваемой прямой круглой трубе при числах Рейнольдса ниже 2000. В экспериментах использовалась медная труба с внутренним диаметром 22 мм и длиной 1500 мм, нагреваемая электрическим током в условиях постоянного теплового потока. Труба оборачивалась несколькими слоями теплоизоляции для предотвращения потерь тепла. Показано, что в условиях меньшего значения теплового потока на стенке теплоперенос ухудшается вследствие вызванного подъемной силой снижения температурного градиента на внутренней поверхности трубы. Найдено также, что даже в диапазоне чисел Рейнольдса 500–800 при большом значении теплового потока наблюдается увеличение теплопереноса, обусловленное колебаниями скорости из-за нестационарности течения при появлении точки перегиба в профиле скоростей, вызванной сильным действием подъемной силы вблизи стенки.